

# FRTB's P&L attribution test

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work based on publication co-authored  
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# Presentation overview

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## **Context**

- Fundamental Review of the Trading Book
  - where do things stand today?
  - how did we get here?
  - where does the P&L attribution test sit within FRTB
- Motivation for our work on the P&L Attribution test

## ***Presentation of our work on the P&L Attribution test***

- Assumptions, mathematics, etc.
- Results, implications of those results

## ***Where from here?***

- Possible solutions
- Lessons

## FRTB has stalled

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- Basically radio silence from Basel for the past twelve months
- Changing of the guard within Basel's (renamed) Market Risk Group
- Singapore, Hong Kong, Malaysia, Japan, Europe, USA, Canada, Australia  
→ regulators across jurisdictions are aligned in *postponing implementation*  
→ ...none of them especially keen to put hard deadlines on the table
- "Diminished hopes" for *any semblance* of a level playing field outcome...
- Baby, bathwater ??? →

*"None of us are against good, sound principles, but we must realise the idea of a level playing field is a myth."*

Andrew Sheng

*Chief adviser to the China Banking Regulatory Commission*

# So why has FRTB stalled?

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## Internal Model Approach (IMA)

- P&L Attribution (PLA) test
- Non-Modellable Risk Factor (NMRF) charge
- Expected Shortfall calculation
  - Computationally onerous
  - Reduced vs Full risk factor coverage – unclear how to demonstrate.
- So onerous it effectively dis-incentivises any bank currently on SA to even consider

## Standardised Approach (SA)

- Ill-conceived treatment of curvature risk, basis risk
- Computationally onerous, esp. if calculated *daily* (e.g. Curvature calculations)
- ...so onerous that Basel has *already* proposed a *Simplified Standardised Approach*
- Will be used as a floor to the IMA charge – but no one is quite sure how...
- Least of its problems is that it's punitively calibrated

## And it's partly the *industry's* fault...

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## How did we get here?

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- Industry too slow to *really* engage in the formulation process
  - Draft 1: First Basel consultation paper: May 2012
  - Draft 2: Second consultation: October 2013
  - Draft 3: Third consultation: December 2014
  - Draft 4: Instruction for Basel Monitoring: February 2015
  - Draft 5: Quantitative Impact Study: July 2015
  - Draft 6: “Final” release of Market risk standard: January 2016

... and ***from the beginning*** Basel was upfront in asking for industry to constructively engage.

- *When did you, or your bank, seriously start looking at FRTB?*
- Has the lack of timely, constructive feedback from industry been mistaken by the Basel Committee as tacit endorsement for their proposals?

## What is the PLA Test?

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- New **desk-level** test to complement Backtesting as part of the framework for use of the **Internal Model Approach** (IMA) for market risk capital
- Monthly test, designed to check how closely the *daily* P&L calculated by the front-office tracks the *daily* P&L calculated by Risk
- More broadly ties back to the question of how representative Risk's projected P&L distribution might be of the actual P&L distribution  
→ Is the IMA loss metric (VaR, ES) appropriate to set adequate capital?
- ?? Tangential aim of making credible the threat of pushing Pillar 1 capital to a Standardised basis when internal models perform poorly (eg., GFC)

*There needs to be some sort of "punishing" process associated with having poor Internal Models.*

Member of Basel's MRG, meeting in Ottawa, 10th October 2017  
(from ISDA's minutes of the meeting)



## PLA test - Definition

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- Monthly check of two ratios,  $MS$  and  $VV$ , against prescribed thresholds:

$$MS : \quad \rho_{MS} = \left| \frac{m_E}{S_H} \right| \leq 10\% \quad E = (R - H) = \text{Unexplained (Error) P\&L}$$
$$VV : \quad \rho_{VV} = \frac{S^2_E}{S^2_H} \leq 20\% \quad R = \text{Risk-theoretical P\&L}$$
$$H = \text{Hypothetical P\&L}$$

- Month is a Fail if *either* ratio exceeds its threshold
- 4<sup>th</sup> Failed month** in a rolling 12-month window is a FAIL of the PLA test

$MS = \text{Mean/Standard deviation}$

$VV = \text{Variance/Variance}$



## Consequences of failing the PLA test

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- Desk loses accreditation to use IMA, and must revert to using SA.
- No “Ifs” or “Buts”: the result is mathematical and *unequivocal*
- No “traffic light” escalation, or any chance to remediate (cf. backtesting)
- No decision or engagement required from the prudential regulator
- No consideration of the tens of millions of dollars the bank might have spent pursuing IMA accreditation, and the cottage industries that have been built up within banks to support it
- **This should have been an obvious warning sign that the PLA test had not been properly thought through by Basel regulators.**

# Background to our work

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## **Motivation**

- ANZ's own preliminary results indicated high failure rates
- Portfolios which had *low* P&L volatility had the *highest* failure rates  
→ started to sniff a rat with the *mathematical definition* of the PLA test
- Partly motivated by a casual suggestion from a regulator that “*with big enough desks, any noise should just diversify away – so what's the issue?*”

## **Optics**

- The *optics* of having an author outside the industry was important.
- PLA test is academic, ivory-tower regulation.  
Perhaps a critique from within the *same tower* would be more effective?

# Mathematical assumptions for ANZ's analysis

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- Assume that for an individual instrument on day  $i$ , both  $H_i$  and  $E_i$  are random normal variables, with a *relative* variance of  $\sigma^2$

$$H_i \sim N(0,1) \quad E_i = (H_i - R_i) \sim N(0, \sigma^2)$$

- Assume that across all  $n$  distinct instruments in the desk,  $H_i$ 's are correlated  $\gamma_H$ ,  $E_i$ 's are correlated  $\gamma_E$  (both  $\gamma_H, \gamma_E$  constant)
- Therefore for the desk of  $n$  distinct instruments on day  $i$ ,

$$H_{n,i} \sim N(0, q_H) \quad E_{n,i} \sim N(0, \sigma^2 \cdot q_E) \quad q_* = n + \gamma_*(n^2 - n)$$

## Maths (cont'd)

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$$m_{E_n} = \frac{1}{21} \sum_{i=1}^{21} E_{n,i}$$

$$s^2_{E_n} = \frac{1}{20} \sum_{i=1}^{21} (E_{n,i} - m_{E_n})^2$$

$$m_{E_n} \sim N\left(0, \frac{\sigma^2 \cdot q_E}{21}\right)$$

$$s^2_{E_n} \sim \chi^2_{20} \cdot \frac{\sigma^2 \cdot q_E}{20}$$

$$\sqrt{\frac{21}{\sigma^2} \cdot \frac{q_H}{q_E}} \cdot \rho_{MS} \sim t_{20}$$

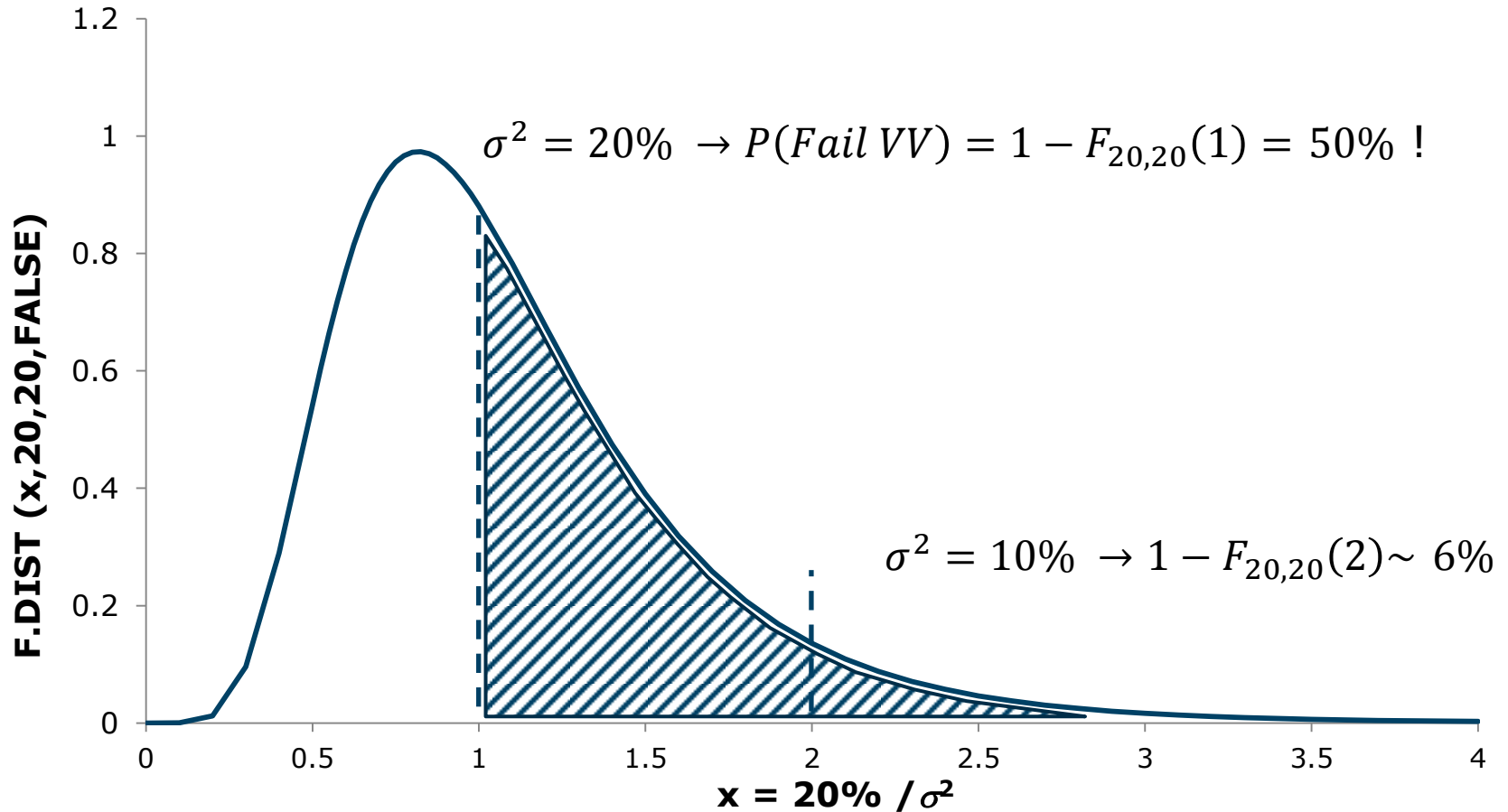
$$\frac{1}{\sigma^2} \cdot \frac{q_H}{q_E} \cdot \rho_{VV} \sim F_{20,20}$$

Student t-distn

Fisher's F-distn

# Fisher's F-distribution is the problem here

$$P(\text{Fail } VV) = P(\rho_{VV} > 20\%) = 1 - F_{20,20}\left(\frac{20\%}{\sigma^2}\right)$$

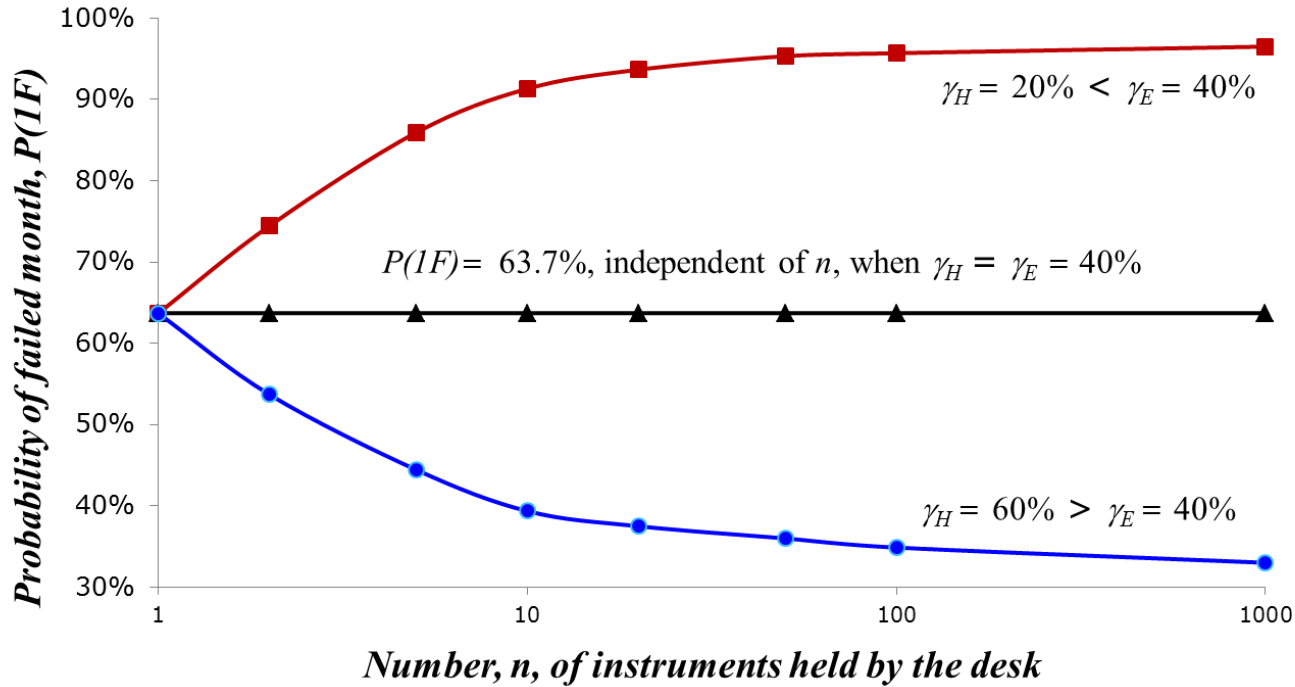


## Probability that *month* is a fail

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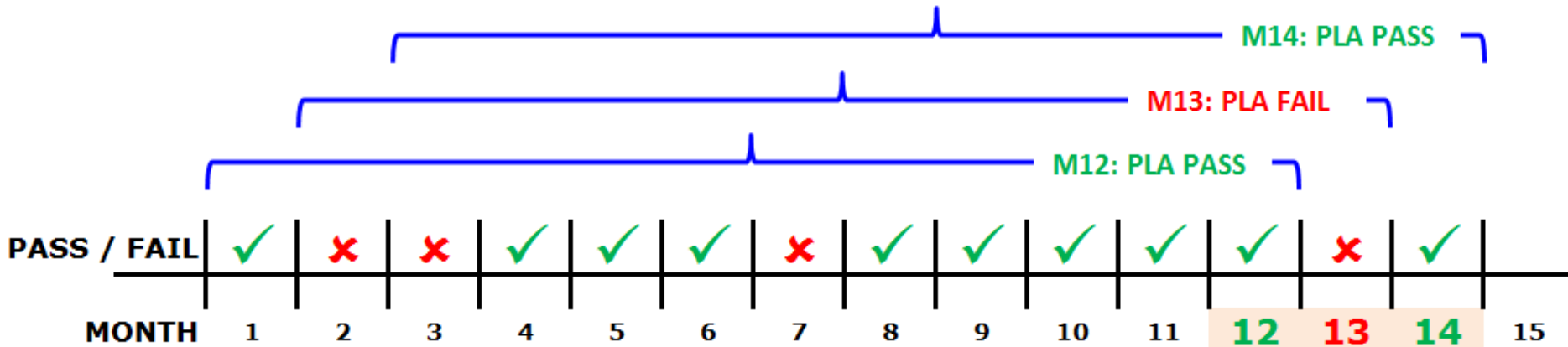
$\rho_{VV} = \sigma^2$	$P(\text{Fail } VV)$	$P(\text{Fail } MS)$	$P(1F)$ <i>(month is a fail)</i>
30%	81.4%	41.3%	87.8%
28%	77.1%	39.7%	84.7%
26%	71.9%	37.9%	80.8%
24%	65.6%	36.1%	76.1%
22%	58.3%	34.0%	70.5%
20%	50.0%	31.8%	63.7%
18%	40.8%	29.3%	56.0%
16%	31.1%	26.5%	47.4%
14%	21.6%	23.5%	38.3%
12%	13.1%	20.1%	29.3%
10%	6.5%	16.3%	21.0%
8%	2.3%	12.1%	13.8%
6%	0.5%	7.6%	8.0%
4%	0.0%	3.3%	3.3%
2%	0.0%	0.4%	0.4%

# Relaxing the assumptions around correlation



$\gamma_E = 40\%$		Number of instruments in desk					
		$n=2$	$n=5$	$n=10$	$n=20$	$n=50$	$n=100$
$\gamma_H$	0%	84.2%	99.0%	100.0%	100.0%	100.0%	100.0%
	20%	73.7%	86.2%	91.6%	93.5%	95.2%	95.5%
	40%	64.5%	63.8%	63.7%	63.9%	63.6%	63.4%
	60%	53.1%	44.3%	39.8%	37.8%	36.3%	35.3%
	80%	45.7%	30.5%	25.9%	23.3%	21.9%	21.2%
	100%	38.3%	22.6%	17.7%	15.9%	14.0%	13.9%

# PLA FAIL is FOUR failed months in a ROLLING YEAR



- Probability of surviving *next month* (“survival probability”) :

$$P(A | B_{<4})$$

$A$  = Probability that next month both the MS and VV ratios PASS

$B_{<4}$  = Probability that prior twelve months has *less than 4* failed months

$$P(A | B_{<4}) = \frac{1}{P(B_{<4})} \cdot [P(B_0) + P(B_1) + P(B_2) + P(B_{3,1F}) + P(1P) \cdot P(B_{3,1P})]$$



# Probabilities of failing the PLA test

$\rho_{VV} = \sigma^2$	<i>P (Fail PLA) before 1 year</i>	<i>P (Fail PLA) within subsequent...</i>			
		1 year	2 years	3 years	4 years
30%	100%	100%	100%	100%	100%
28%	100%	100%	100%	100%	100%
26%	100%	100%	100%	100%	100%
24%	100%	100%	100%	100%	100%
22%	99.8%	99.9%	100%	100%	100%
20%	99.3%	99.8%	100%	100%	100%
18%	97.0%	99.2%	100%	100%	100%
16%	89.9%	97.0%	100%	100%	100%
14%	73.7%	90.1%	99.0%	99.9%	100%
12%	48.6%	73.4%	92.9%	98.1%	99.5%
10%	23.1%	45.9%	70.7%	84.1%	91.4%
8%	7.2%	18.6%	33.7%	46.0%	56.0%
6%	1.19%	3.77%	7.40%	10.90%	14.26%
4%	0.049%	0.178%	0.355%	0.532%	0.709%
2%	0.000014%	0.000054%	0.000108%	0.000162%	0.000216%

Getting on the horse is *hard*

Falling off the horse is *easy*



# Multiple desks

- Report published by the ECB on 29 September 2017
- A **third** of surveyed banks estimate having **more than 50** trading desks !



Working group on the implementation of the FRTB within the SSM

**Industry conference call on FRTB**

Svenja Schenkel  
Jan Busse  
Karsten Stickelmann

Frankfurt, 29 September 2017

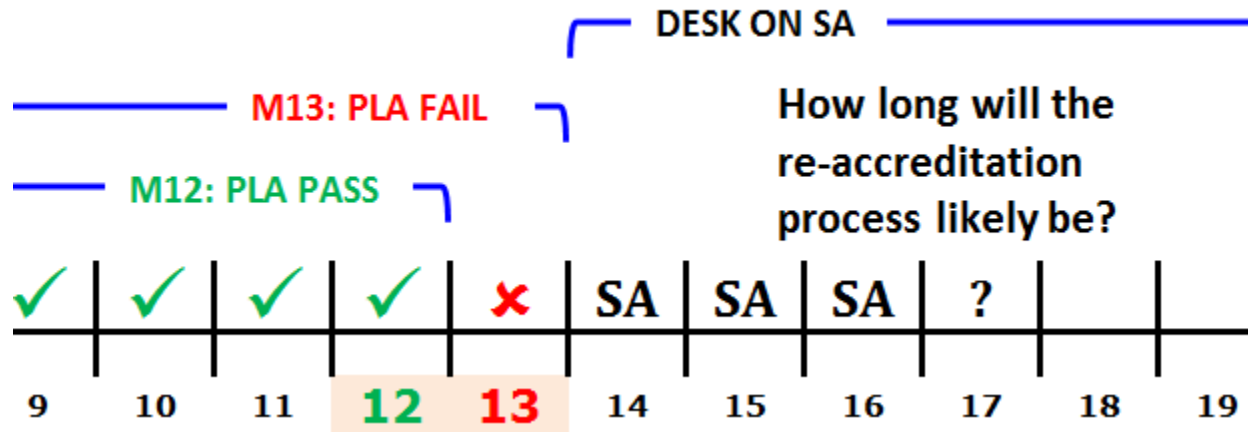
## Results

- The currently estimated number of FRTB trading desks (TDs) per bank varies strongly from <10 to >100:

# of desks	% of banks
≤10	32%
≤30	32%
≤50	0%
≤70	23%
≤90	5%
>90	9%

- 39% of the banks planning to apply for IMA approval envisage including all desks in their internal model; the remainder envisage an IMA approval for just a subset of trading desks.

# Expected steady-state proportion of desks on IMA



- Depends on how long *realistically* for the process of remediation and *then* becoming re-accredited by the regulator to use IMA...



# Steady state solution

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- Assume that remediation + re-accreditation from regulator takes **k**-months
- Equate *prior* to *posterior* probabilities, to get steady-state IMA desk proportion

$$\pi_{IMA} = \pi_{IMA} \cdot P(A | B_{<4}) + \pi_{SA,k}$$

Proportion of desks  
on IMA *next* month

=

Proportion of desks  
on IMA and which  
pass PLA *this* month

+

Proportion of desks on SA  
which failed PLA k-months  
ago but which will become  
reaccredited *next* month

$$\pi_{SA,2} = \pi_{SA,1}$$

⋮

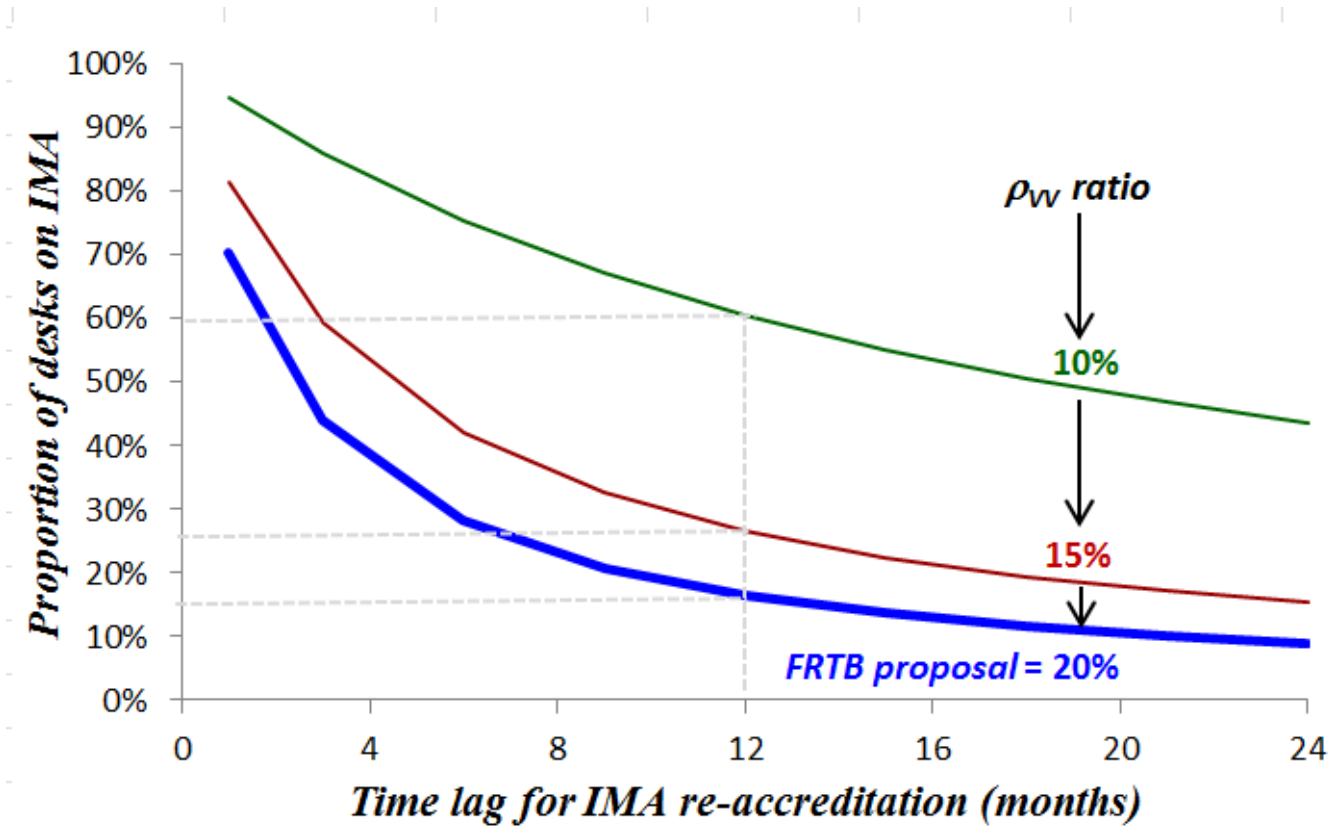
$$\pi_{SA,k} = \pi_{SA,k-1}$$

Subject to  $\pi_{IMA} + \sum_i^k \pi_{SA,i} = 1$

$$\rightarrow \pi_{IMA} = \frac{1}{1 + k(1 - P(A | B_{<4}))}$$

# Steady-state proportion of desks on IMA

- Is having *more than half* of desks on IMA, at any time, optimistic ?



## What can be done to help?

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- Relative variance of unexplained P&L compared with Hypothetical P&L needs to be a *low single digit percentage* (say  $\sigma^2 < 5\%$ ), for all desks.  
→ that, *by itself*, will be a fairly tough ask for most desks to achieve
- *Average* correlation of Hypo P&L between instruments should be consistently greater than *average* correlation of Unexplained P&L  
→ May (?) be possible to orchestrate, over reasonable periods of time (?)  
→ increased Hypo P&L correlation → *increased chance* Backtesting failures
- Need a short turn-around time for re-accreditation, subsequent to failure.  
→ Probably naïve to think this would be less than 6 months. Or even 12?  
→ Do regulators *really* want to be at the middle of that merry-go-round?

## Can the PLA test be salvaged ?

- If it *has* to be a *monthly* test, then ANZ's view is that there is no chance. One month (~21 days) of data is just not enough
  - underlying distributions just too broad
  - too much noise in such small sample size
- ISDA still trying. Results of their "beauty contest" show no clear preference for an alternative
- ISDA proposing *further 2 years*, beyond the implementation date, for parallel testing of PLA test

Proposals	Count of 1st preference	Count of 2nd preference	Count of 3rd preference
Normalised BCBS metrics	5	5	4
Spearman Correlation Test plus Kolmogorov-Smirnov Test	9	7	5
Symmetrical Variance Ratio	7	7	8
Ratio of Expected Shortfall	3	2	6
Ratio of Expected Shortfall + Symmetrical Variance Ratio	2	2	2
Direct Volatility Test	1	6	2
Tail Correlation	3	1	2
Stressed PLA	1	1	1
<b>Total</b>	31	31	30

*"[Is] it possible the industry proposals result in replacing one bad test with another bad test?"*

Member of Basel's MRG, meeting in Ottawa, 10<sup>th</sup> October 2017

# What is the behaviour that PLA seeks to encourage?

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- The underlying concern of regulators is really with *model validation*. How can that be addressed *without* something like the PLA test?

My personal answer:

→ in *much more sensible ways*, starting with Pillar 2

- Risk that regulators are encouraging banks to take their eye *off* the ball?  
Three 'spinning plates' to juggle at the core of the FRTB :
  - PLA test
  - Non-Modellable Risk Factor charge
  - Backtesting
- Which of the three above *most directly* pertains to the assessment and allocation of prudential levels of loss absorbing capital?



## Conclusions

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- PLA test is fundamentally flawed. Implementation will be difficult
- One of the intractable problems with FRTB as currently drafted
- The lack of focussed engagement from the industry over the past four years has been totally lamentable.
- Lesson here is that you can't wait for the "final" version to be on the table *and only then* swing into action to have a closer look.

## QUESTIONS