FRTB’s P&L attribution test

Peter Thompson

work based on publication co-authored by Hayden Luo and Kevin Fergusson

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Views expressed herein are my own and not those of ANZ
Presentation overview

**Context**

- Fundamental Review of the Trading Book
  - where do things stand today?
  - how did we get here?
  - where does the P&L attribution test sit within FRTB

- Motivation for our work on the P&L Attribution test

*Presentation of our work on the P&L Attribution test*

- Assumptions, mathematics, etc.
- Results, implications of those results

*Where from here?*

- Possible solutions
- Lessons
FRTB has stalled

- Basically radio silence from Basel for the past twelve months
- Changing of the guard within Basel’s (renamed) Market Risk Group
- Singapore, Hong Kong, Malaysia, Japan, Europe, USA, Canada, Australia → regulators across jurisdictions are aligned in *postponing implementation*
  → ...none of them especially keen to put hard deadlines on the table

- “Diminished hopes” for *any semblance* of a level playing field outcome...

- Baby, bathwater ??? → “None of us are against good, sound principles, but we must realise the idea of a level playing field is a myth.”

  Andrew Sheng
  *Chief adviser to the China Banking Regulatory Commission*
So why has FRTB stalled?

Internal Model Approach (IMA)

- P&L Attribution (PLA) test
- Non-Modellable Risk Factor (NMRF) charge
- Expected Shortfall calculation
  - Computationally onerous
  - Reduced vs Full risk factor coverage – unclear how to demonstrate.
- So onerous it effectively dis-incentivises any bank currently on SA to even consider

Standardised Approach (SA)

- Ill-conceived treatment of curvature risk, basis risk
- Computationally onerous, esp. if calculated daily (e.g. Curvature calculations)
- ...so onerous that Basel has already proposed a Simplified Standardised Approach
- Will be used as a floor to the IMA charge – but no one is quite sure how...
- Least of its problems is that it’s punitively calibrated
And it’s partly the *industry’s* fault...
How did we get here?

• Industry too slow to really engage in the formulation process
  → Draft 1: First Basel consultation paper: May 2012
  → Draft 2: Second consultation: October 2013
  → Draft 3: Third consultation: December 2014
  → Draft 4: Instruction for Basel Monitoring: February 2015

... and from the beginning Basel was upfront in asking for industry to constructively engage.

• When did you, or your bank, seriously start looking at FRTB?

• Has the lack of timely, constructive feedback from industry been mistaken by the Basel Committee as tacit endorsement for their proposals?
What is the PLA Test?

- New **desk-level** test to complement Backtesting as part of the framework for use of the **Internal Model Approach** (IMA) for market risk capital

- Monthly test, designed to check how closely the *daily* P&L calculated by the front-office tracks the *daily* P&L calculated by Risk

- More broadly ties back to the question of how representative Risk’s projected P&L distribution might be of the actual P&L distribution → Is the IMA loss metric (VaR, ES) appropriate to set *adequate capital*?

- ?? Tangential aim of making credible the threat of pushing Pillar 1 capital to a Standardised basis when internal models perform poorly (eg., GFC)

*There needs to be some sort of “punishing” process associated with having poor Internal Models.*

Member of Basel’s MRG, meeting in Ottawa, 10th October 2017
(from ISDA’s minutes of the meeting)
PLA test - Definition

- Monthly check of two ratios, $MS$ and $VV$, against prescribed thresholds:

$$MS : \quad \rho_{MS} = \left| \frac{m_E}{s_H} \right| \leq 10\%$$

$$VV : \quad \rho_{VV} = \frac{s^2_E}{s^2_H} \leq 20\%$$

- Month is a Fail if *either* ratio exceeds its threshold

- 4th Failed month in a rolling 12-month window is a FAIL of the PLA test

$MS = \text{Mean/Standard deviation} \quad VV = \text{Variance/Variance}$
Consequences of failing the PLA test

• Desk loses accreditation to use IMA, and must revert to using SA.

• No “Ifs” or “Buts”: the result is mathematical and *unequivocal*

• No “traffic light” escalation, or any chance to remediate (cf. backtesting)

• No decision or engagement required from the prudential regulator

• No consideration of the tens of millions of dollars the bank might have spent pursuing IMA accreditation, and the cottage industries that have been built up within banks to support it

• *This should have been an obvious warning sign that the PLA test had not been properly thought through by Basel regulators.*
Background to our work

Motivation

- ANZ’s own preliminary results indicated high failure rates
- Portfolios which had low P&L volatility had the highest failure rates
  → started to sniff a rat with the mathematical definition of the PLA test
- Partly motivated by a casual suggestion from a regulator that “with big enough desks, any noise should just diversify away – so what’s the issue?”

Optics

- The optics of having an author outside the industry was important.
- PLA test is academic, ivory-tower regulation.
  Perhaps a critique from within the same tower would be more effective?
Mathematical assumptions for ANZ’s analysis

- Assume that for an individual instrument on day \(i\), both \(H_i\) and \(E_i\) are random normal variables, with a *relative* variance of \(\sigma^2\)

\[
H_i \sim N(0,1) \quad E_i = (H_i - R_i) \sim N(0, \sigma^2)
\]

- Assume that across all \(n\) distinct instruments in the desk, \(H_i\)'s are correlated \(\gamma_H\), \(E_i\)'s are correlated \(\gamma_E\) (both \(\gamma_H, \gamma_E\) constant)

- Therefore for the desk of \(n\) distinct instruments on day \(i\),

\[
H_{n,i} \sim N(0, q_{H}) \quad E_{n,i} \sim N(0, \sigma^2.q_{E}) \quad q_* = n + \gamma_*(n^2 - n)
\]
Maths (cont’d)

\[ m_{En} = \frac{1}{21} \sum_{i=1}^{21} E_{n,i} \]

\[ s^2_{En} = \frac{1}{20} \sum_{i=1}^{21} (E_{n,i} - m_{En})^2 \]

\[ m_{En} \sim N \left( 0, \frac{\sigma^2 \cdot q_E}{21} \right) \]

\[ s^2_{En} \sim \chi^2_{20} \cdot \frac{\sigma^2 \cdot q_E}{20} \]

\[ \sqrt{\frac{21 \cdot q_H}{\sigma^2 \cdot q_E}} \cdot \rho_{MS} \sim t_{20} \]

\[ \frac{1}{\sigma^2 \cdot q_E} \cdot \rho_{VV} \sim F_{20,20} \]

Student t-distn  
Fisher’s F-distn
Fisher’s F-distribution is the problem here

\[ P(\text{Fail } VV) = P(\rho_{VV} > 20\%) = 1 - F_{20,20} \left( \frac{20\%}{\sigma^2} \right) \]

\[ \sigma^2 = 20\% \rightarrow P(\text{Fail } VV) = 1 - F_{20,20}(1) = 50\% ! \]

\[ \sigma^2 = 10\% \rightarrow 1 - F_{20,20}(2) \sim 6\% \]
## Probability that *month* is a fail

| $\rho_{VV} = \sigma^2$ | $P(\text{Fail VV})$ | $P(\text{Fail MS})$ | $P(1F)$
|------------------------|----------------------|---------------------|-----------------------
| 30%                    | 81.4%                | 41.3%               | 87.8%                 |
| 28%                    | 77.1%                | 39.7%               | 84.7%                 |
| 26%                    | 71.9%                | 37.9%               | 80.8%                 |
| 24%                    | 65.6%                | 36.1%               | 76.1%                 |
| 22%                    | 58.3%                | 34.0%               | 70.5%                 |
| 20%                    | 50.0%                | 31.8%               | 63.7%                 |
| 18%                    | 40.8%                | 29.3%               | 56.0%                 |
| 16%                    | 31.1%                | 26.5%               | 47.4%                 |
| 14%                    | 21.6%                | 23.5%               | 38.3%                 |
| 12%                    | 13.1%                | 20.1%               | 29.3%                 |
| 10%                    | 6.5%                 | 16.3%               | 21.0%                 |
| 8%                     | 2.3%                 | 12.1%               | 13.8%                 |
| 6%                     | 0.5%                 | 7.6%                | 8.0%                  |
| 4%                     | 0.0%                 | 3.3%                | 3.3%                  |
| 2%                     | 0.0%                 | 0.4%                | 0.4%                  |
Relaxing the assumptions around correlation

$$\gamma_H = 20\% < \gamma_E = 40\%$$

$$\gamma_H = 60\% > \gamma_E = 40\%$$

Probability of failed month, $P(1F)$

Number, $n$, of instruments held by the desk

<table>
<thead>
<tr>
<th>$\gamma_E = 40%$</th>
<th>Number of instruments in desk</th>
<th>$n=2$</th>
<th>$n=5$</th>
<th>$n=10$</th>
<th>$n=20$</th>
<th>$n=50$</th>
<th>$n=100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_H$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0%</td>
<td></td>
<td>84.2%</td>
<td>99.0%</td>
<td>100.0%</td>
<td>100.0%</td>
<td>100.0%</td>
<td>100.0%</td>
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<tr>
<td>20%</td>
<td></td>
<td>73.7%</td>
<td>86.2%</td>
<td>91.6%</td>
<td>93.5%</td>
<td>95.2%</td>
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<td>64.5%</td>
<td>63.8%</td>
<td>63.7%</td>
<td>63.9%</td>
<td>63.6%</td>
<td>63.4%</td>
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<tr>
<td>60%</td>
<td></td>
<td>53.1%</td>
<td>44.3%</td>
<td>39.8%</td>
<td>37.8%</td>
<td>36.3%</td>
<td>35.3%</td>
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<tr>
<td>80%</td>
<td></td>
<td>45.7%</td>
<td>30.5%</td>
<td>25.9%</td>
<td>23.3%</td>
<td>21.9%</td>
<td>21.2%</td>
</tr>
<tr>
<td>100%</td>
<td></td>
<td>38.3%</td>
<td>22.6%</td>
<td>17.7%</td>
<td>15.9%</td>
<td>14.0%</td>
<td>13.9%</td>
</tr>
</tbody>
</table>
PLA FAIL is FOUR failed months in a ROLLING YEAR

- Probability of surviving next month ("survival probability"):

  \[ P(A \mid B_{<4}) \]

  \[ A = \text{Probability that next month both the MS and VV ratios PASS} \]

  \[ B_{<4} = \text{Probability that prior twelve months has less than 4 failed months} \]

  \[ P(A \mid B_{<4}) = \frac{1}{P(B_{<4})} \cdot \left[ P(B_0) + P(B_1) + P(B_2) + P(B_{3,1F}) + P(1P).P(B_{3,1P}) \right] \]
Probabilities of failing the PLA test

<table>
<thead>
<tr>
<th>$\rho_{VV} = \sigma^2$</th>
<th>$P(\text{Fail PLA})$ before 1 year</th>
<th>$P(\text{Fail PLA})$ within subsequent...</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 year</td>
<td>2 years</td>
</tr>
<tr>
<td>30%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>28%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>26%</td>
<td>100%</td>
<td>100%</td>
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<tr>
<td>24%</td>
<td>100%</td>
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<tr>
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<td>99.8%</td>
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<tr>
<td>20%</td>
<td>99.3%</td>
<td>99.8%</td>
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<tr>
<td>18%</td>
<td>97.0%</td>
<td>99.2%</td>
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<tr>
<td>16%</td>
<td>89.9%</td>
<td>97.0%</td>
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<tr>
<td>14%</td>
<td>73.7%</td>
<td>90.1%</td>
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<td>12%</td>
<td>48.6%</td>
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<tr>
<td>10%</td>
<td>23.1%</td>
<td>45.9%</td>
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<tr>
<td>8%</td>
<td>7.2%</td>
<td>18.6%</td>
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<tr>
<td>6%</td>
<td>1.19%</td>
<td>3.77%</td>
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<tr>
<td>4%</td>
<td>0.049%</td>
<td>0.178%</td>
</tr>
<tr>
<td>2%</td>
<td>0.000014%</td>
<td>0.000054%</td>
</tr>
</tbody>
</table>

Getting on the horse is *hard*  
Falling off the horse is *easy*
Multiple desks

- Report published by the ECB on 29 September 2017
- A third of surveyed banks estimate having *more than 50* trading desks!

### Results

- The currently estimated number of FRTB trading desks (TDs) per bank varies strongly from <10 to >100:

<table>
<thead>
<tr>
<th># of desks</th>
<th>% of banks</th>
</tr>
</thead>
<tbody>
<tr>
<td>≤10</td>
<td>32%</td>
</tr>
<tr>
<td>≤30</td>
<td>32%</td>
</tr>
<tr>
<td>≤50</td>
<td>0%</td>
</tr>
<tr>
<td>≤70</td>
<td>23%</td>
</tr>
<tr>
<td>≤90</td>
<td>5%</td>
</tr>
<tr>
<td>&gt;90</td>
<td>9%</td>
</tr>
</tbody>
</table>

- 39% of the banks planning to apply for IMA approval envisage including all desks in their internal model; the remainder envisage an IMA approval for just a subset of trading desks.
Expected steady-state proportion of desks on IMA

- Depends on how long *realistically* for the process of remediation and *then* becoming re-accredited by the regulator to use IMA...
Steady state solution

- Assume that remediation + re-accreditation from regulator takes \( k \)-months
- Equate prior to posterior probabilities, to get steady-state IMA desk proportion

\[
\pi_{IMA} = \pi_{IMA} \cdot P(A \mid B_{<4}) + \pi_{SA,k}
\]

Proportion of desks on IMA next month

Proportion of desks on IMA and which pass PLA this month

Proportion of desks on SA which failed PLA \( k \)-months ago but which will become reaccredited next month

\[
\begin{align*}
\pi_{SA,2} &= \pi_{SA,1} \\
\vdots \\
\pi_{SA,k} &= \pi_{SA,k-1}
\end{align*}
\]

Subject to \( \pi_{IMA} + \sum_{i}^{k} \pi_{SA,i} = 1 \)

\[
\begin{align*}
\rightarrow \\
\pi_{IMA} &= \frac{1}{1 + k(1 - P(A \mid B_{<4}))}
\end{align*}
\]
Steady-state proportion of desks on IMA

- Is having *more than half* of desks on IMA, at any time, optimistic?
What can be done to help?

- Relative variance of unexplained P&L compared with Hypothetical P&L needs to be a low single digit percentage (say \(\sigma^2<5\%\)), for all desks. 
  → that, by itself, will be a fairly tough ask for most desks to achieve

- *Average* correlation of Hypo P&L between instruments should be consistently greater than *average* correlation of Unexplained P&L 
  → May (?) be possible to orchestrate, over reasonable periods of time (?) 
  → increased Hypo P&L correlation → *increased chance* Backtesting failures

- Need a short turn-around time for re-accreditation, subsequent to failure. 
  → Probably naïve to think this would be less than 6 months. Or even 12? 
  → Do regulators *really* want to be at the middle of that merry-go-round?
Can the PLA test be salvaged?

- If it *has* to be a *monthly* test, then ANZ’s view is that there is no chance. One month (~21 days) of data is just not enough
  → underlying distributions just too broad
  → too much noise in such small sample size

- ISDA still trying. Results of their “beauty contest” show no clear preference for an alternative

- ISDA proposing *further 2 years*, beyond the implementation date, for parallel testing of PLA test

<table>
<thead>
<tr>
<th>Proposals</th>
<th>Count of 1st preference</th>
<th>Count of 2nd preference</th>
<th>Count of 3rd preference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalised BCBS metrics</td>
<td>5</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Spearman Correlation Test plus Kolmogorov-Smirnov Test</td>
<td>9</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>Symmetrical Variance Ratio</td>
<td>7</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>Ratio of Expected Shortfall</td>
<td>3</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>Ratio of Expected Shortfall + Symmetrical Variance Ratio</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Direct Volatility Test</td>
<td>1</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>Tail Correlation</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Stressed PLA</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>31</strong></td>
<td><strong>31</strong></td>
<td><strong>30</strong></td>
</tr>
</tbody>
</table>

“*[Is] it possible the industry proposals result in replacing one bad test with another bad test?*”

Member of Basel’s MRG, meeting in Ottawa, 10th October 2017
What is the behaviour that PLA seeks to encourage?

• The underlying concern of regulators is really with *model validation*. How can that be addressed *without* something like the PLA test?

  My personal answer:

  → in *much more sensible ways*, starting with Pillar 2

• Risk that regulators are encouraging banks to take their eye *off* the ball? Three ‘spinning plates’ to juggle at the core of the FRTB:

  → PLA test
  → Non-Modellable Risk Factor charge
  → Backtesting

• Which of the three above *most directly* pertains to the assessment and allocation of prudential levels of loss absorbing capital?
Conclusions

• PLA test is fundamentally flawed. Implementation will be difficult.

• One of the intractable problems with FRTB as currently drafted.

• The lack of focussed engagement from the industry over the past four years has been totally lamentable.

• Lesson here is that you can’t wait for the “final” version to be on the table and only then swing into action to have a closer look.

QUESTIONS