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FRTB

Three Quantitative Curiosities

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FRTB - what we want to discuss

- VaR vs ES (Expected Shortfall) – is it really necessary?
- Liquidity Horizons and Overlapping Observations – are there unintended consequences?
- Mixing Correlation Matrices – is it legitimate?
- Some results from QIS

VaR vs ES

Common Steps:

1. Generate the vector of market perturbations : $\Delta R = (\Delta R_1, \Delta R_2, \dots, \Delta R_n)$ and apply then to the current market rate vector $R = (R_1, R_2, \dots, R_n)$ to generate the perturbed vector $\tilde{R} = R + \Delta R$. Perturbations can also be fractional or mixed,
   Perturbations can be generated by a Monte Carlo simulation or sampled from historical data.

2. For portfolio *P* generate a vector of hypothetical P&Ls:
$$\Delta P(R) = P(\tilde{R}) - P(R) = (\Delta P_1, \Delta P_2, \dots, \Delta P_n)$$

3. $$\boldsymbol{VaR_q(P) = -Quantile_{1-q}(\Delta P(R))}$$
   $$\boldsymbol{ES_q(P) = -E\left(\Delta P(R) | \Delta P(R) < -VaR_q(P)\right)}$$

   ***q*=99%** (VaR) ***q*=97.5%** (ES)

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VaR vs ES

VaR has been in use since the 90s
Academics have been pushing for ES because:

A. It is coherent: $ES(P_1 \cup P_2) \leq ES(P_1) + ES(P_2)$

B. It catches outliers.

C. It potentially takes autocorrelations in the data in to account

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A. VaR is not coherent, but known examples are based on discontinuous P&L vectors. In practice portfolios are well diversified and continuous

B. For a typical 97.5% level and >500 simulation you have 12 observation in the tail, so the impact of the outlier is largely diluted.
Also, VaR is just one risk metrics, Stress Test is designed to catch portoflio non-linearities.
VaR is easier for management and traders to understand

C. We'll have a look this later.
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VaR vs ES – some examples

In terms of the order statistics:

$$VaR(\Delta P(R))_q = - \Delta P(R)_{[(1-q)n]}; \qquad ES(\Delta P(R))_q = - \frac{1}{[(1-q)n]} \sum_{i=1}^{[(1-q)n]} \Delta P(R)_{[i]}$$

As per Basel recommended levels we compare 99% VaR vs 97.5% ES

Distribution	VaR(99%)	ES (97.5%)	Ratio
Normal	-2.341	-2.330	0.996
t(7)	-2.967	-3.002	1.012
t(3)	-4.503	-4.806	1.067
t(2)	-7.386	-8.406	1.14
Cauchy	-33.120	-71.572	2.161

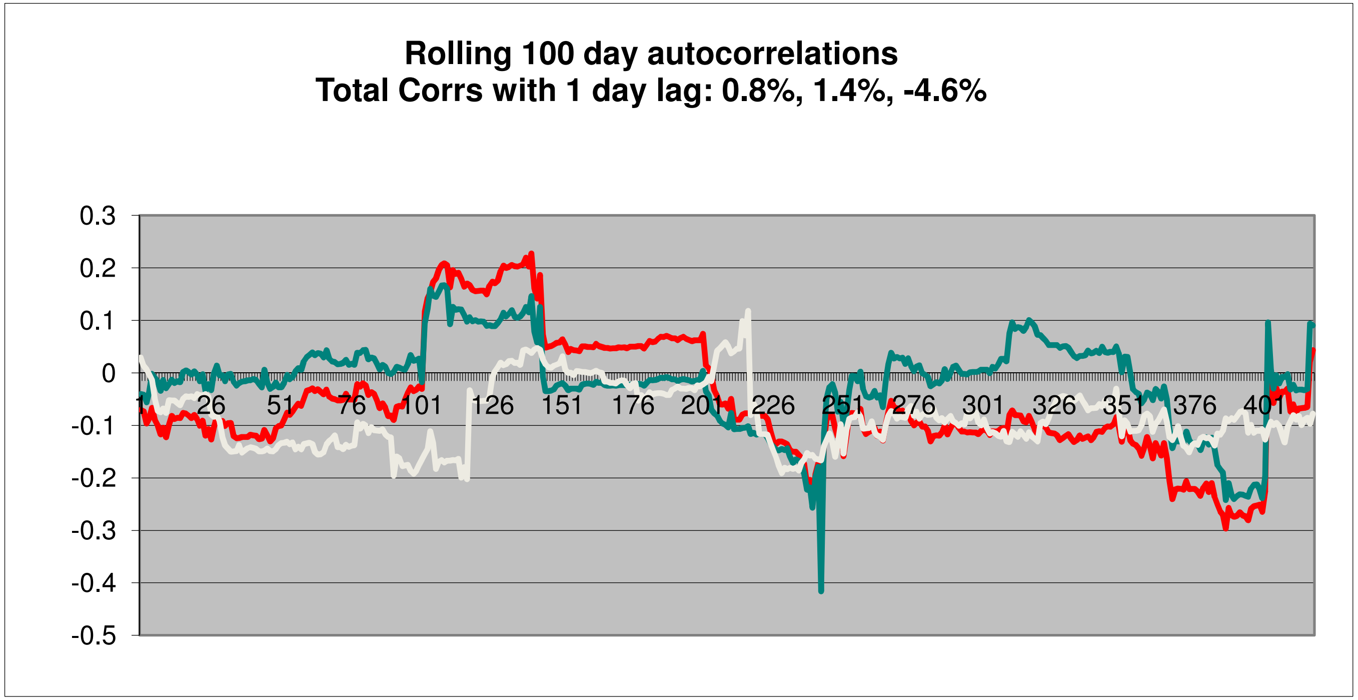
VaR vs ES – more examples.

My point is: What’s the point?

Trade	Var (99%)/ ES (97.5%) Ratio	SVaR (99%)/ SES (97.5%) Ratio
1	0.949	0.989
2	0.983	0.987
3	0.967	1.018
4	0.956	1.035
5	0.981	1.050
6	0.956	1.000
7	1.000	0.985
8	0.977	1.022
9	0.977	0.965
10	1.014	0.997

VaR vs ES - Data Autocorrelation

P&L data typically does not have significant autocorrelation



For longer lags correlation may increase

As we'll see later, modelled ES/VaR ratio does not change if autocorrelation is introduced

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Liquidity Horizons

Motivation:

Time required to unwind the position - .

Current liquidity horizon:

10 (working) days for all asset classes.

Proposed liquidity horizons:

10, 20, 60, 120, 250 days depending on the asset class

Problem:

The sample is not long enough to generate IID of the required length. Even if it was, the question of 'The Second I' - *Identical* would arise

Two solutions:

- Use overlapping sums
- Use scaling by $\sqrt{LH}$, implicitly assuming the IID and normality of the perturbations.

Liquidity Horizons

Mathematically

Let $X = (x_1, x_2, \ldots, x_n)$, define $Y = (y_1, y_2, \ldots, x_{n-m})$ where

$$y_i = \sum_{j=i}^{i+m-1} x_j$$ and m is the liquidity horizon

Then calculate VaR and ES on Y as usual

Some mathematics is possible, but we do mathematical experimentation using Monte Carlo

References:
Frishling V, Lauer M, Some Properties of the 10 Day Rolling VaR Estimate, QMF 2007
Sun H, Nelken I, Han G, Guo J, Error of VAR by overlapping intervals, Risk Magazine, Mar 2009

Liquidity Horizons – some results

Distribution\Liquidity Horizon	1	10	20	60	120	250
Normal						
VaR(99%)	2.34	7.40	10.40	17.90	24.96	36.51
Scaled VaR	2.34	7.40	10.47	18.13	25.64	37.01
VaR Stderr	0.16	0.27	0.65	1.13	1.26	2.40
Overlapping VaR	2.34	6.93	9.25	13.79	14.15	14.74
Overlapping VaR Stderr	0.16	0.92	1.81	4.65	7.19	13.08
ES(97.5%)	2.33	7.39	10.35	17.81	24.94	36.52
Overlapping ES	2.33	6.91	9.16	13.70	13.99	14.68
Overlapping ES Stderr	0.16	0.88	1.76	4.58	7.13	13.08

Distribution\Liquidity Horizon	1	10	20	60	120	250
t(3)						
VaR(99%)	4.56	13.48	18.45	30.57	44.34	65.80
Scaled VaR	4.56	14.43	20.40	35.33	49.97	72.13
VaR Stderr	0.66	2.05	1.98	2.88	3.09	6.17
Overlapping VaR	0.66	12.68	16.28	23.21	23.99	24.89
Overlapping VaR Stderr	0.66	2.78	4.09	6.29	10.27	20.23
ES(97.5%)	4.88	13.65	18.67	31.28	44.33	65.34
Overlapping ES	4.88	12.48	16.02	22.89	23.64	24.81
Overlapping ES Stderr	0.73	2.67	3.87	6.33	10.15	20.23

Liquidity Horizons – more results

Distribution\Liquidity Horizon	1	10	20	60	120	250
t(3), autocorr = 0.3						
VaR(99%)	4.56	18.57	25.97	43.56	63.01	93.77
Scaled VaR	4.56	14.43	20.40	35.33	49.97	72.13
VaR Stderr	0.66	2.83	2.66	4.06	4.48	8.99
Overlapping VaR	4.56	17.48	22.77	32.65	33.72	34.96
Overlapping VaR Stderr	0.66	4.01	5.93	9.11	14.63	28.95
ES(97.5%)	4.88	19.01	26.21	44.49	63.12	93.25
Overlapping ES	4.88	17.15	22.37	32.28	33.22	34.81
Overlapping ES Stderr	0.73	3.84	5.59	9.04	14.48	28.89

Distribution\Liquiduty Horizon	1	10	20	60	120	250
t(2)						
VaR(99%)	7.39	34.56	50.30	89.28	133.34	202.32
Scaled VaR	7.39	23.36	33.03	57.21	80.91	116.79
VaR Stderr	1.53	8.05	9.34	16.05	20.69	26.66
Overlapping VaR	7.39	34.45	41.46	53.12	55.71	59.32
Overlapping VaR Stderr	1.53	15.41	17.35	18.79	21.24	39.55
ES(97.5%)	8.41	38.04	56.09	100.19	154.32	235.39
Overlapping ES	8.41	32.81	40.68	53.12	54.75	58.74
Overlapping ES Stderr	2.05	13.80	17.22	18.79	21.28	39.67

Mixing Correlation Matrices

This is an excerpt from the Basel consultative document :

98. The discounted net cash flows at each vertex are then put into the following formula, which recognises offsetting between cash flows at different vertices in the same currency:

$$K_b = \sqrt{\sum_i RW_i^2 MV_i^2 + \sum_i \sum_{j \neq i} \rho_{ij} RW_i \ MV_i \ W_j \ MV_j} = \sqrt{\text{VRV}}$$

Where MVi is the present value of the net cash flow at vertex i, RWi is the risk weight assigned to vertex i, and ρij is the correlation parameter between vertices i and j, *V = RWxMV, R - correlation matrix*

99. The first correlation matrix below should be used for ρij if the net cash flows at vertices i and j have the same sign (long/long or short/short). The second correlation matrix below should be used if the net cash flows at vertices i and j have different signs (long/short).

This approach (Cashflow based) is now abandoned in favour of the Sensitivity Based Approach

Mixing Correlation Matrices

	0.25y	0.5y	1y	2y	3y	5y	10y	15y	20y	30y
0.25y	100%	95%	85%	75%	65%	55%	45%	40%	40%	35%
0.5y	95%	100%	90%	75%	70%	65%	50%	45%	45%	40%
1y	85%	90%	100%	90%	85%	75%	60%	50%	50%	50%
2y	75%	75%	90%	100%	95%	90%	75%	65%	60%	60%
3y	65%	70%	85%	95%	100%	95%	80%	75%	70%	65%
5y	55%	65%	75%	90%	95%	100%	90%	85%	75%	70%
10y	45%	50%	60%	75%	80%	90%	100%	95%	80%	85%
15y	40%	45%	50%	65%	75%	85%	95%	100%	100%	100%
20y	40%	45%	50%	60%	70%	75%	80%	100%	100%	100%
30y	35%	40%	50%	60%	65%	70%	85%	100%	100%	100%

	0.25y	0.5y	1y	2y	3y	5y	10y	15y	20y	30y
0.25y	100%	90%	70%	55%	50%	40%	35%	20%	15%	15%
0.5y	90%	100%	85%	70%	60%	45%	35%	25%	20%	15%
1y	70%	85%	100%	80%	75%	60%	45%	35%	30%	20%
2y	55%	70%	80%	100%	90%	75%	55%	40%	40%	40%
3y	50%	60%	75%	90%	100%	85%	60%	50%	50%	45%
5y	40%	45%	60%	75%	85%	100%	75%	60%	60%	50%
10y	35%	35%	45%	55%	60%	75%	100%	85%	75%	65%
15y	20%	25%	35%	40%	50%	60%	85%	100%	85%	70%
20y	15%	20%	30%	40%	50%	60%	75%	85%	100%	70%
30y	15%	15%	20%	40%	45%	50%	65%	70%	70%	100%

Mixing Correlation Matrices

Guess what!!! The first matrix is not even positive-semidefinite

Eigen Values:

First matrix eigenvalues:

7.41	1.75	0.52	0.18	0.11	0.09	0.03	0.00	-0.04	-0.05

Second matrix eigenvalues:

5.90	2.15	0.71	0.41	0.29	0.20	0.12	0.11	0.07	0.04

Mixed matrix eigenvalues:

6.72	1.86	0.73	0.51	0.38	0.31	0.02	-0.02	-0.17	-0.33

QIS – Quantitative Impact Study

Initial Proposal: Cash Flow Based method
Significant push back

Revised approach: Sensitivity Based Approach (SBA)

$$\textit{Capital Charge Based on} = \rho\ \textit{GlobalDiversified}: LA - ES + (1-\rho)\begin{bmatrix} LA - ES(FX) \\ +LA - ES\ (IR) \\ +LA - ES(Credit) \\ +LA - ES(Vol) \\ +LA - ES(Commods) \end{bmatrix}$$

LA-ES Liquidity Adjusted Expected Shortfall

QIS - Quantitative Impact Study

Some results

	LAES/[Current 10d 3(VAR+SVAR)]	SBA/LAES
Long 10y/Short2y bonds	77%	65%
IR Swap	64%	99%
CDS Position	120%	148%
Bond/CDS Position	109%	116%
ITRAXX	122%	212%
IR Portfolio	72%	82%
Credit Portfolio	143%	159%